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MECHANICAL ENGINEERING

CONVENTIONAL PRACTICE SETS VOLUME - III

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THEORY OF MACHINES

CONVENTIONAL PRACTICE SETS

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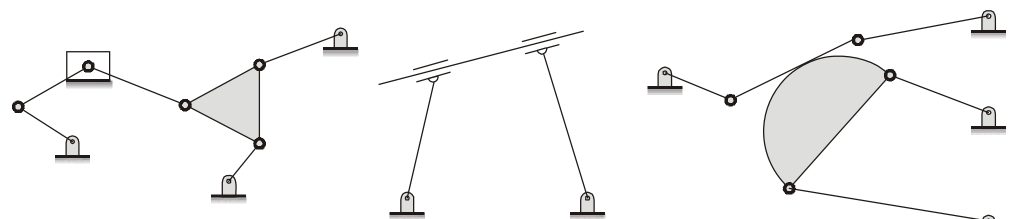
1 CHAPTER Simple Mechanisms

Practice Questions : Level-I

Q.1 Determine the degree of freedom of the mechanisms shown in below figure.



(a) (b) (c)

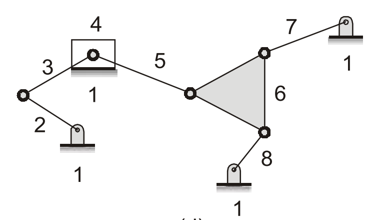
Solution:

(a) The mechanism has a sliding pair. Therefore, its degree of freedom must be found from Grubler's criterion.
Total number of links - 8 (Figure (b))
Number of pairs with 1 degree of freedom = 10
(At the slider, one sliding pair and two turning pairs)

$$\boxed{F = 3(N-1) - 2P_1 - P_2}$$

$$= 3(8-1) - 2 \times 10 - 0 = 1$$

Thus, it is a mechanism with a single degree of freedom.



(d)

(b) The system has a redundant degree of freedom as the rod of the mechanism can slide without causing any movement in the rest of the mechanism.
∴ Effective degree of freedom

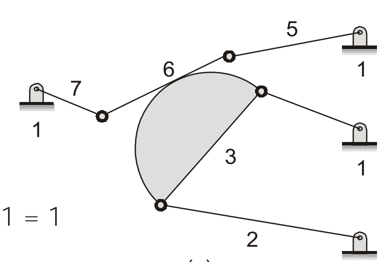
$$= \boxed{3(N-1) - 2P_1 - P_2 - F_r} = 3(4-1) - 2 \times 4 - 0 - 1 = 0$$

As the effective degree of freedom is zero, it is a locked system.

(c) The mechanism has a cam pair. Therefore, its degree of freedom must be found from Grubler's criterion.
Total number of links = 7 (Fig. (e))
Number of pairs with 1 degree of freedom = 8
Number of pairs with 2 degrees of freedom = 1

$$\boxed{F = 3(N-1) - 2P_1 - P_2} = 3(7-1) - 2 \times 8 - 1 = 1$$

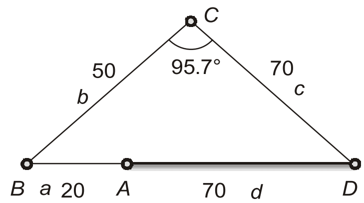
Thus, it is a mechanism with one degree of freedom.



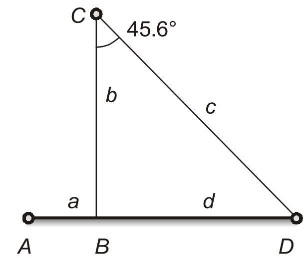
(e)

Q.2 A crank-rocker mechanism has a 70 mm fixed link, a 20 mm crank, a 50 mm coupler, and a 70 mm rocker. Draw the mechanism and determine the maximum and minimum values of the transmission angle. Locate the two toggle positions and find the corresponding crank angles and the transmission angles.

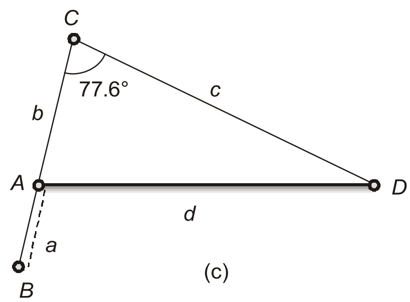
Solution:



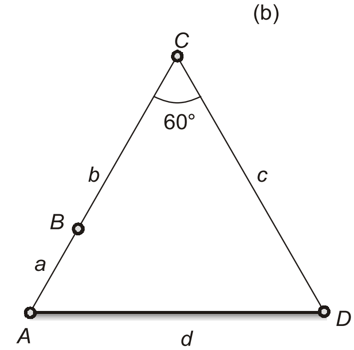
(a)
(mm)



(b)



(c)



(d)

Given data: In this mechanism, Length of the longest link = 70 mm; Length of the shortest link = 20 mm; Length of other links = 70 and 50 mm

Since 70 + 20 < 70 + 50, it belongs to the class-I mechanism. In this case as the link adjacent to the shortest link is fixed, it is a crank-rocker mechanism.

Maximum transmission angle is when θ is 180° [Fig. (a)]

Thus
$$(a + d)^2 = b^2 + c^2 - 2bc\cos\mu$$
$$(20 + 70)^2 = 50^2 + 70^2 - 2 \times 50 \times 70\cos\mu$$
$$8100 = 2500 + 4900 - 7000\cos\mu$$
$$\cos\mu = -0.1$$
$$\mu = 95.7°$$

Minimum transmission angle is when θ is 0°. [Fig. (b)]

Thus
$$(70 - 20)^2 = 50^2 + 70^2 - 2 \times 50 \times 70\cos\mu$$
$$2500 = 2500 + 4900 - 7000\cos\mu$$
$$\mu = 45.6°$$

The two toggle positions are shown in Fig. (c) and (d).

Transmission angle for first position,
$$d^2 = (b - a)^2 + c^2 - 2(b - a)c\cos\mu$$
$$70^2 = 30^2 + 70^2 - 2 \times 30 \times 70\cos\mu$$
$$4900 = 900 + 4900 - 4200\cos\mu$$
$$\cos\mu = 0.214$$
$$\mu = 77.6°$$

As c and d are of equal length [Fig. (c)], it is an isosceles triangle and thus input angle θ = (77.6° + 180°) = 257.6°

Transmission angle for second position Fig. (d),
$$d^2 = (b + a)^2 + c^2 - 2(b + a)c\cos\mu$$
$$70^2 = 70^2 + 70^2 - 2 \times 70 \times 70\cos\mu$$
$$4900 = 4900 + 4900 - 9800\cos\mu$$
$$\cos\mu = 0.5$$
$$\mu = 60°$$

(or as all the sides of the triangle of Fig. (d) are of equal length, it is an equilateral triangle and thus transmission angle is equal to 60°)

And the input angle, $\theta = 60°$

- The above results can also be obtained graphically by drawing the figures to scale and measuring the angles.

Q.3 In a crank and slotted lever quick return motion mechanism, the distance between the fixed centres is 240 mm and the length of the driving crank is 120 mm. Find the inclination of the slotted bar with the vertical in the extreme position and the time ratio of cutting stroke to the return stroke.

If the length of the slotted bar is 450 mm, find the length of the stroke if the line of stroke passes through the extreme positions of the free end of the lever.

Solution:

Given data: $AC = 240$ mm; $CB_1 = 120$ mm; $AP_1 = 450$ mm

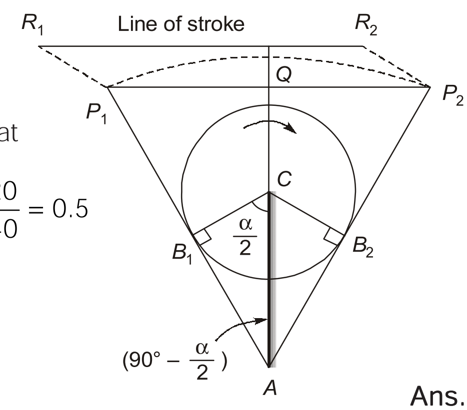


Figure 5

Inclination of the slotted bar with the vertical

Let, $\angle CAB_1$ = Inclination of the slotted bar with the vertical.

The extreme positions of the crank are shown in Fig. 5. We know that

$$\sin \angle CAB_1 = \sin\left(90° - \frac{\alpha}{2}\right) = \frac{B_1C}{AC} = \frac{120}{240} = 0.5$$

$$\therefore \quad \angle CAB_1 = 90° - \frac{\alpha}{2}$$

$$= \sin^{-1}0.5 = 30° \qquad \text{Ans.}$$

Time ratio of cutting stroke to the return stroke

We know that

$$90° - \alpha/2 = 30°$$

$$\therefore \quad \alpha/2 = 90° - 30° = 60°$$

$$\text{or} \quad \alpha = 2 \times 60° = 120°$$

$$\therefore \quad \boxed{\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360° - \alpha}{\alpha}} = \frac{360° - 120°}{120°} = 2 \qquad \text{Ans.}$$

Length of the stroke

We know that length of the stroke,

$$R_1R_2 = P_1P_2 = 2P_1Q$$

$$= 2AP_1 \sin(90° - \alpha/2)$$

$$= 2 \times 450 \sin(90° - 60°)$$

$$= 900 \times 0.5 = 450 \text{ mm} \qquad \text{Ans.}$$

Q.4 In a Whitworth quick return motion mechanism, as shown in Fig. 6, the distance between the fixed centers is 50 mm and the length of the driving crank is 75 mm. The length of the slotted lever is 150 mm and the length of the connecting rod is 135 mm. Find the ratio of the time of cutting stroke to the time of return stroke and also the effective stroke.

Solution:

Given data: $CD = 50$ mm; $CA = 75$ mm; $PA = 150$ mm; $PR = 135$ mm

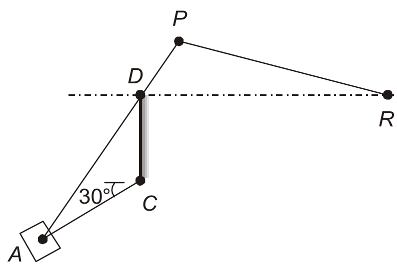


Figure 6

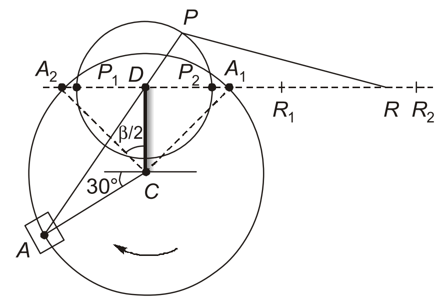


Figure 7

The extreme positions of the driving crank are shown in Fig. 7. From the geometry of the figure,

$$\cos \beta/2 = \frac{CD}{CA_2} = \frac{50}{75} = 0.667 \qquad ...(\because\ CA_2 = CA)$$

$$\beta/2 = 48.2 \quad \text{or} \quad \beta = 96.4°$$

Ratio of the time of cutting stroke to the time of return stroke

We know that

$$\boxed{\frac{\text{Time of cutting stroke}}{\text{Time of return stroke}} = \frac{360-\beta}{\beta}} = \frac{360-96.4}{96.4} = 2.735 \qquad \textbf{Ans.}$$

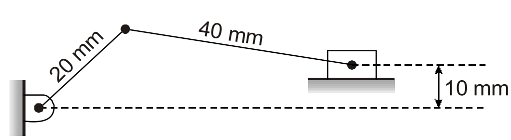
Length of effective stroke

In order to find the length of effective stroke (i.e. R_1R_2), draw the space diagram of the mechanism to some suitable scale, as shown in Fig. 7. Mark $P_1R_2 = P_2R_2 = PR$. Therefore by measurement we find that,

Length of effective stroke = R_1R_2 = 87.5 mm. **Ans.**

Practice Questions : Level-II

Q.5 An offset slider-crank mechanism is shown in the figure at an instant. Conventionally, the Quick Return Ratio (QRR) is considered to be greater than one. What is the value of QRR?



Solution:

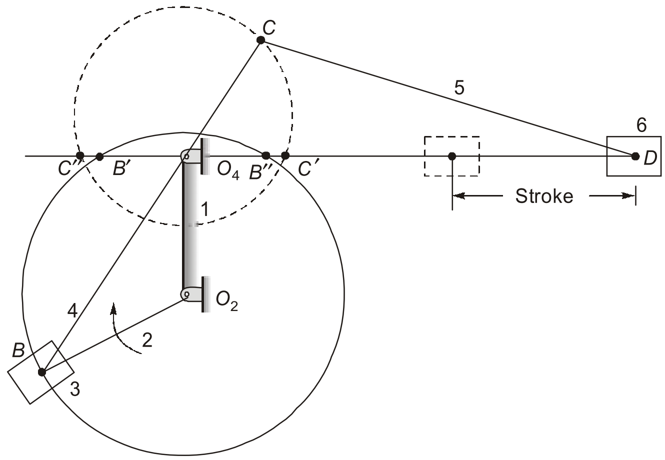
Given data: Length of connecting rod, l_r = 40 mm; Crank radius, r = 20 mm; Eccentricity, e = 10 mm

$$\boxed{\phi = \cos^{-1}\left(\frac{e}{l+r}\right) - \cos^{-1}\left(\frac{e}{l-r}\right)} = \cos^{-1}\left(\frac{1}{6}\right) - \cos^{-1}\left(\frac{1}{2}\right) = 20.4°$$

$$\text{QRR} = \frac{\text{Time of advance stroke}}{\text{Time of return stroke}} = \left(\frac{\theta_C}{2\pi N}\right)\left(\frac{2\pi N}{\theta_R}\right)$$

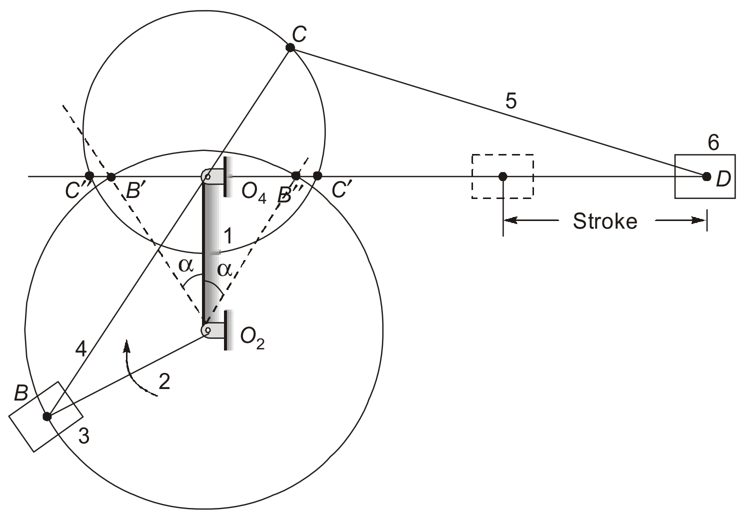
$$\text{QRR} = \frac{\theta_C}{\theta_R} = \frac{180° + \phi}{180° - \phi} = \frac{180° + 20.4°}{180° - 20.4°} = 1.25$$

Q.6 Design a Whitworth quick return mechanism as shown in the figure. Driving crank is to rotate clockwise at constant speed and the time ratio is 2 : 1. Length O_2O_4 is 76.2 mm and the length of stroke is 343 mm. Further assume CD = $3O_4C$. Note pivot O_2 is placed below the pivot O_4. Compute the required values of O_2B, O_4C and CD.



Solution:

As per given information:



$$\text{Time ratio (quick return ration)} = \frac{360 - 2\alpha}{2\alpha} = 2 \quad \Rightarrow \alpha = 60^\circ$$

$$\cos\alpha = \frac{O_2O_4}{O_2B'}$$

$$\cos 60^\circ = \frac{76.2}{O_2B'}$$

$$O_2B' = \frac{76.2}{\cos 60^\circ} = 152.4 \text{ mm} = O_2B$$

Length of the driving crank = 152.4 mm

Length the stroke = $343 = 2 \times O_4C$

O_4C = 171.5 mm

As per given condition CD = $3 \times O_4C = 3 \times 171.5$ mm = 514.5 mm

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STRENGTH OF MATERIALS

CONVENTIONAL PRACTICE SETS

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Simple Stress, Strain and Elastic Constants

Q.1 A prismatic circular bar of diameter 20 mm and length 2.8 m is subjected to a tensile force of 10 kN. The measured extension of the bar is 1.2 mm. Calculate the tensile stress and strain in the bar.

Solution:

$$\text{Tensile stress } (\sigma) = \frac{P}{\text{Area}} = \frac{10 \times 10^3}{\frac{\pi}{4}(20)^2} = 31.83 \text{ N/mm}^2 = 31.83 \text{ MPa}$$

$$\text{Tensile strain } (\in) = \frac{\Delta L}{L} = \frac{1.2}{2800} = 4.286 \times 10^{-4}$$

Q.2 In the above question find the actual or true stress (σ_a).

Solution:

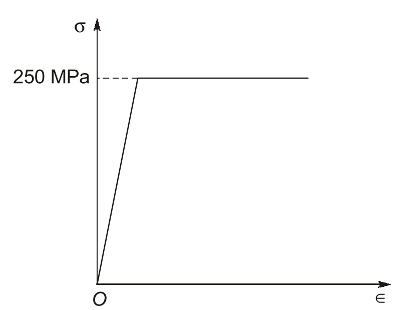
We know that for tension,

$$\boxed{\text{Actual stress } (\sigma_a) = \sigma(1+\in)}$$

$$= 31.83\,(1 + 4.285 \times 10^{-4})$$

$$= 31.84 \text{ N/mm}^2 = 31.84 \text{ MPa}$$

Q.3 A bar of length 2 m is made of mild steel for which idealized stress-strain curve is shown. The yield stress of material of bar is 250 MPa and the slope of curve is 200 GPa. The bar is loaded axially until it elongates 6.5 mm. Find the final length of the bar after removal of the load.



Solution:

$$\text{Length } (L_0) = 2000 \text{ mm}$$

$$\text{Yield strain} = \text{elastic strain} = \frac{\sigma_y}{E} = \frac{250}{2 \times 10^5} = 1.25 \times 10^{-3}$$

$$\text{Total strain on loading} = \frac{6.5}{2000} = 3.25 \times 10^{-3}$$

Since total strain is greater than elastic strain, it means loading is applied beyond elastic limit. Therefore after unloading, certain permanent plastic strain will be left in the bar.

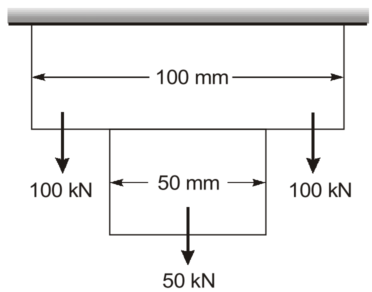
Plastic strain ($\in_P$) $= 3.25 \times 10^{-3} - 1.25 \times 10^{-3} = 2 \times 10^{-3}$

Permanent deformation after unloading

$$= \in_P \times L_0$$
$$= 2 \times 10^{-3} \times 2000 = 4 \text{ mm}$$

Final length after unloading $= 2000 + 4 = 2004$ mm

Q.4 A bar of varying square cross-section is loaded symmetrically as shown in the figure. Loads shown are placed on one of the axes of symmetry of cross-section. Ignoring self weight, calculate the maximum tensile stress anywhere in the section

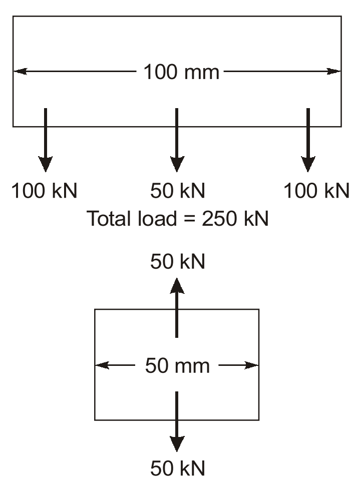


Solution:

$$\text{The stress in lower bar} = \frac{50 \times 1000}{50 \times 50} = 20 \text{ N/mm}^2 = 20 \text{ MPa}$$

$$\text{The stress in upper bar} = \frac{250 \times 1000}{100 \times 100} = 25 \text{ N/mm}^2 = 25 \text{ MPa}$$

Thus the maximum tensile stress any where in the bar is 25 N/mm^2.



Q.5 A metal bar of length 100 mm is inserted between two rigid supports and its temperature is increased by 10°C. If the coefficient of thermal expansion is 12×10^{-6} per °C and the Young's modulus is 2×10^5 MPa, then calculate the stress in the bar.

Solution:

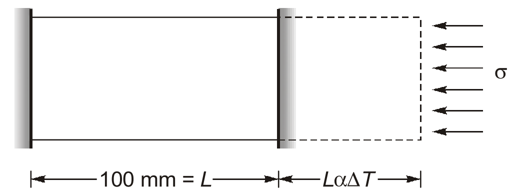
Given $\alpha = 12 \times 10^{-6}$/°C, $E = 2 \times 10^5$ MPa, $\Delta T = 10$°C

$$\boxed{\text{Temperature stress} = \alpha TE}$$
$$= 12 \times 10^{-6} \times 10 \times 2 \times 10^5 = 24 \text{ MPa}$$

Due to temperature,

$$\Delta L = L\alpha\Delta T$$

But since support in fixed so, expansion is not allowed so stress is developed in the bar which is compressive in nature.



Now,

Expansion due to temperature = Compression due to stress

$$L\alpha\Delta T = \frac{\sigma}{E}\times L$$

Thermal stress, $\sigma = E\alpha\Delta T$

$$= 1\times10^5\times12\times10^{-6}\times10$$

$$= 24\text{ MPa}$$

Q.6 A mild steel specimen is under uniaxial tensile stress. Young's modulus and yield stress for mild steel are 2×10^5 MPa and 250 MPa respectively. Calculate the maximum amount of strain energy per unit volume that can be stored in this specimen without permanent deformation.

Solution:

The strain energy per unit volume may be given as

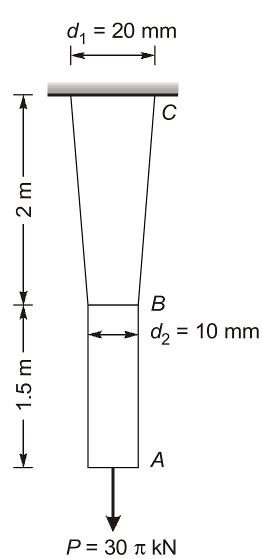
$$u = \frac{1}{2}\times\text{ Stress }\times\text{Strain} = \frac{1}{2}\times\text{Stress}\times\frac{\text{Stress}}{E} = \frac{1}{2}\frac{(\text{Stress})^2}{E}$$

For maximum strain energy without permanent deformation, stress = yield stress (σ_y)

$$u = \frac{1}{2}\times\frac{\sigma_y^2}{E} = \frac{1}{2}\times\frac{(250)^2}{2\times10^5}$$

$$= 0.156\text{ N-mm/mm}^3$$

Q.7 A tapered circular rod of diameter varying from 20 mm to 10 mm is connected to another uniform circular rod of diameter 10 mm as shown in the following figure. Both bars are made of same material with the modulus of elasticity, $E = 2\times10^5$ MPa. If load subjected is 30π kN, then calculate deflection at point A.



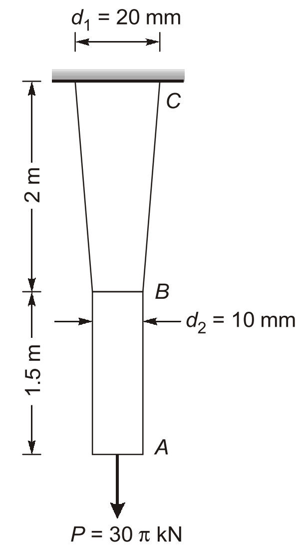
Solution:

$P = 30\pi \times 10^3$ N, $L_1 = 1.5$ m, $d_1 = 10$ mm, $L_2 = 2$ m, $d_2 = 20$ mm

AB is uniform

So, $$\Delta_{AB} = \frac{PL_1}{A_1 E}$$

BC is tapered



$$\boxed{\Delta_{BC} = \frac{PL_2}{\frac{\pi}{4} d_1 d_2 E}}$$

Total elongation, $$\Delta = \Delta_{AB} + \Delta_{BC}$$

$$= \frac{PL_1}{A_1 E} + \frac{4PL_2}{\pi d_1 d_2 E}$$

$$= \frac{30\pi \times 10^3 \times 1.5 \times 10^3}{\frac{\pi}{4} \times (10)^2 \times 2 \times 10^5} + + \frac{30\pi \times 10^3 \times 2 \times 10^3}{\frac{\pi}{4} \times 10 \times 20 \times 2 \times 10^5}$$

$$= (9 + 6) \text{ mm} = 15 \text{ mm}$$

Q.8 A steel specimen of 12 mm diameter extends by 6.31×10^{-2} mm over a gauge length of 150 mm when subjected to an axial load of 10 kN. The same specimen undergoes a twist of 0.5° an a length of 150 mm over a twisting moment of 10 N-m. Using the above data, determine the elastic constants E, μ, G and K.

Solution:

Tensile Test; Axial load, $P = 10$ kN

Length of specimen, $L = 150$ mm

Cross-sectional area, $A = \frac{\pi}{4} \times 12^2 = 113.09 \text{ mm}^2$

Change in length of specimen, $\Delta = 6.31 \times 10^{-2}$ mm

Let E N/mm² is modulus of elasticity of material.

We know, axial deformation due to axial load is given by

$$\Delta = \frac{PL}{AE}$$

$$\therefore \quad E = \frac{PL}{A\Delta} = \frac{10 \times 1000 \times 150}{113.09 \times 6.31 \times 10^{-2}} = 2.10 \times 10^5 \text{ N/mm}^2$$

Torsion test:

We know, $$\frac{T}{I_P} = \frac{G\theta}{L} \quad \text{...(i)}$$

∴ Modulus of rigidity, $$G = \frac{TL}{I_P \theta}$$

Polar moment of inertia, $$I_P = \frac{\pi}{32} D^4 = \frac{\pi}{32} \times (12)^4 = 2035.75 \text{ mm}^4$$

Angle of twist, $\theta = \dfrac{0.5\times\pi}{180}$ radian $= 8.73 \times 10^{-3}$ radian

From eq. (i), we get

$$G = \frac{10\times10^3\times150}{2035.75\times8.73\times10^{-3}}$$
$$= 8.44 \times 10^4\ \text{N/mm}^2$$

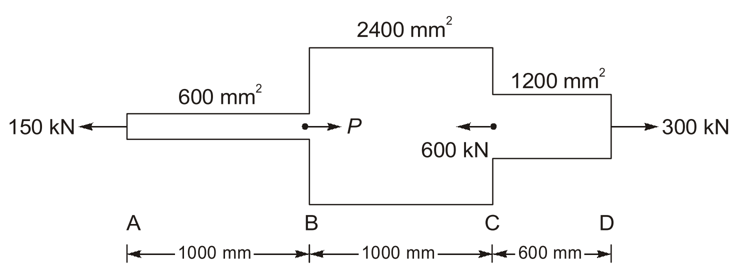
We know, $\boxed{E = 2G(1+\mu)}$

$$\frac{E}{2G} = 1 + \mu$$

$\therefore$
$$\mu = \frac{E}{2G} - 1 = \frac{2.10\times10^5}{2\times8.44\times10^4} - 1 = 1.24 - 1$$
$$= 0.24$$

Also
$$E = 3K(1 - 2\mu)$$
$$K = \frac{E}{3(1-2\mu)} = \frac{2.10\times10^5}{3(1-2\times0.24)}$$
$$= 1.35 \times 10^5\ \text{N/mm}^2$$

Q.9 A member *ABCD* is subjected to concentrated loads as shown. Calculate
(i) Force *P* necessary for equilibrium
(ii) Total elongation of bar
The value of Young's modulus of elasticity is given as $E = 2 \times 10^5$ N/mm^2

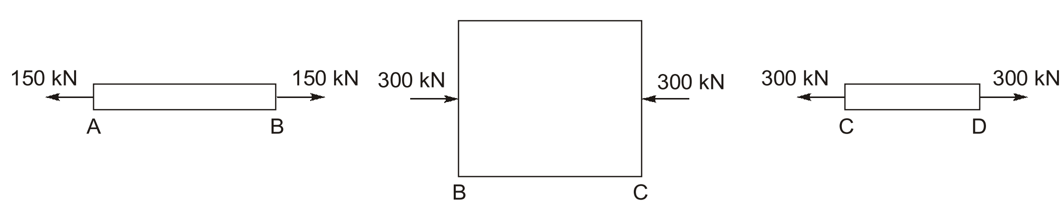


Solution:

(i) For equilibrium, $\Sigma F_H = 0$

$$(P + 300) - (150 + 600) = 0$$
$$P = 450\ \text{kN}$$

(ii)



$$\Delta_{\text{Total}} = \Delta_{AB} + \Delta_{BC} + \Delta_{CD}$$
$$= \frac{150\times1000\times1000}{600\times2\times10^5} - \frac{300\times1000\times1000}{2400\times2\times10^5} + \frac{300\times600\times1000}{1200\times2\times10^5}$$
$$= 1.25 - 0.625 + 0.75$$
$$= 1.375\ \text{mm (elongation)}$$

MACHINE DESIGN

CONVENTIONAL PRACTICE SETS

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1 CHAPTER Design Against Fluctuating Load

Practice Questions : Level-I

Q.1 During a high cycle fatigue test, a metallic specimen is subjected to cyclic loading with a mean stress of +140 MPa, and a minimum stress of –70 MPa. Determine the R-ratio (minimum stress to maximum stress) for this cyclic loading.

Solution:

$$\sigma_{min} = -70 \text{ MPa}$$

$$\sigma_{mean} = 140 \text{ MPa}$$

$$\frac{\sigma_{min}}{\sigma_{max}} = ?$$

We know that,

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$140 = \frac{\sigma_{max} - 70}{2}$$

$$280 + 70 = \sigma_{max}$$

$$\sigma_{max} = 350 \text{ MPa (Tensile)}$$

$$\frac{\sigma_{min}}{\sigma_{max}} = -\frac{70}{350} = -0.2$$

Q.2 Fatigue life of a material for a fully reversed loading condition is estimated from $\sigma_a = 1100\, N^{-0.15}$ where σ_a is the stress amplitude in MPa and N is the failure life in cycles. Determine the maximum allowable stress amplitude for a life of 1×10^5 cycles under the same loading condition.

Solution:

$$\text{Stress amplitude } (\sigma_a) = \frac{\sigma_{max} - \sigma_{min}}{2} = 1100\ N^{-0.15}$$

$$\frac{\sigma_{max} - (-\sigma_{max})}{2} = 1100\, N^{-0.15}$$

[∵ for reversal loading $\sigma_{max} = -\sigma_{min}$]

$$\frac{2\sigma_{max}}{2} = 1100\, N^{-0.15}$$

$$\sigma_{max} = 1100\ N^{-0.15} = 1100 \times (10^5)^{-0.15}$$

$$= 1100 \times (10)^{-0.75}$$

$$= \frac{1100}{5.62}$$

$$\boldsymbol{\sigma_{max} = 195.61\ \text{MPa}}$$

Q.3 A small element at the critical section of a component in biaxial state of stress with the two principal stresses being 360 MPa and 140 MPa. Determine the maximum working stress according to distortion energy theory.

Solution:

Given: σ_1 = 360 MPa, σ_2 = 140 MPa

According to distortion energy theory,

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2} = 314 \text{ MPa}$$

Q.4 A machine component made of a ductile material is subjected to a variable loading with σ_{min} = – 50 MPa and σ_{max} = 50 MPa. If the corrected endurance limit and the yield strength for the material are σ_e' = 100 MPa and σ_y = 300 MPa, respectively, determine the factor of safety.

Solution:

Variable loading is a completely reversed fatigue or variable loading because

$$\sigma_{max} = -\sigma_{min}$$

Hence, $\sigma_{mean} = \sigma_m = 0$; $\sigma_{variable} = \sigma_v = \sigma_{max}$

For completely reversed fatigue loading Soderberg, Goodman, Gerber and strength criterion will give same results.

As per strength criterion,

$$\sigma_{max} \leq \sigma_{per} \quad \text{or} \quad \frac{\text{Failure stress}}{\text{F.O.S.}}$$

$$\sigma_{max} = \frac{\text{Endurance limit}}{N}$$

$$N = \frac{\text{Endurance limit}}{\text{Maximum stress}} = \frac{100}{50} = 2$$

Q.5 A thin spherical pressure vessel of 200 mm diameter and 1 mm thickness is subjected to an internal pressure varying from 4 to 8 MPa. Assume that the yield, ultimate, and endurance strength of material are 600, 800 and 400 MPa respectively. Find the factor of safety as per Goodman's relation.

Solution:

Stress induced, $$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

$$\sigma_{1max} = \frac{8 \times 100}{2 \times 1} = 400 \text{ MPa}$$

$$\sigma_{1min} = \frac{4 \times 100}{2 \times 1} = 200 \text{ MPa}$$

$$\sigma_{2max} = 400 \text{ MPa}$$

$$\sigma_{2min} = 200 \text{ MPa}$$

$$\sigma_{1m} = \frac{\sigma_{1max} + \sigma_{1min}}{2} = 300 \text{ MPa}$$

$$\sigma_{1a} = \frac{\sigma_{1max} - \sigma_{1min}}{2} = 100 \text{ MPa}$$

$$\sigma_{2m} = 300 \text{ MPa}$$

$$\sigma_{2a} = 100 \text{ MPa}$$

Equivalent Stresses

$$\sigma_{me} = \sqrt{\sigma_{1m}^2 + \sigma_{2m}^2 - \sigma_{1m}\sigma_{2m}} = \sqrt{300^2 + 300^2 - 300 \times 300} = 300 \text{ MPa}$$

Similarly,

$$\sigma_{ae} = \sqrt{\sigma_{1a}^2 + \sigma_{2a}^2 - \sigma_{1a}\sigma_{2a}} = 100 \text{ MPa}$$

Goodman equation,

$$\frac{\sigma_{ae}}{S_e} + \frac{\sigma_{me}}{S_{ut}} = \frac{1}{N}$$

$$\Rightarrow \quad \frac{100}{400} + \frac{300}{800} = \frac{1}{N}$$

$$N = 1.6$$

Q.6 A forged steel link with uniform diameter of 30 mm at the centre is subjected to an axial force that varies from 40 kN in compression to 160 kN in tension. The tensile (S_u), yield (S_y) and corrected endurance (S_e) strengths of the steel material are 600 MPa, 420 MPa and 240 MPa respectively. Determine the factor of safety against fatigue endurance as per Soderberg's criterion.

Solution:

Diameter, $d = 30$ mm

$F_{max} = +160$ kN (Tension)

$F_{min} = -40$ kN (Compression)

Tensile strength, $S_u = 600$ MPa

Yield strength, $S_y = 420$ MPa

Corrected endurance, $S_e = 240$ MPa

Maximum stress, $$\sigma_{max} = \frac{F_{max}}{A} = \frac{160 \times 10^3 \text{ N}}{\frac{\pi}{4}(30)^2 \text{ mm}^2} = 226.47 \text{ MPa (Tensile)}$$

Minimum stress, $$\sigma_{min} = \frac{F_{min}}{A} = \frac{-40 \times 10^3 \text{ N}}{\frac{\pi}{4}(30)^2 \text{ mm}^2} = -56.62 \text{ MPa (Compression)}$$

Stress amplitude, $$\sigma_a = \frac{1}{2}(\sigma_{max} - \sigma_{min}) = \frac{1}{2}[226.47 - (-56.62)] = 141.54 \text{ MPa}$$

Mean stress, $$\sigma_m = \frac{1}{2}(\sigma_{max} + \sigma_{min}) = \frac{1}{2}[226.47 + (-56.62)] = 84.925 \text{ MPa}$$

Assume factor of safety is n then

$$S_a = N\sigma_a = 141.54\, N$$

$$S_m = N\sigma_m = 84.925\, N$$

The equation of Soderberg line is as follows

$$\frac{S_a}{S_e} + \frac{S_m}{S_{yt}} = 1$$

$$\Rightarrow \quad \frac{141.54\, N}{240} + \frac{84.925\, N}{420} = 1$$

$$\Rightarrow \quad n = 1.26$$

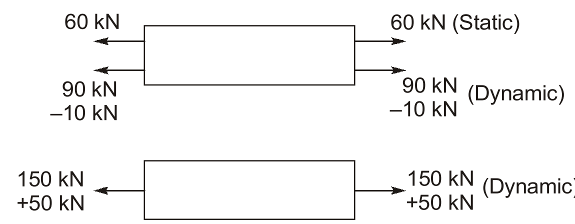
Q.7 A bar is subjected to a combination of a steady load of 60 kN and a load fluctuating between –10 kN and 90 kN. The corrected endurance limit of the bar is 150 MPa. the yield strength of the material is 480 MPa and the ultimate strength of the material is 600 MPa. The bar cross-section is square with side *a*. If the factor of safety is 2, determine the value of *a* (in mm), according to the modified Goodman's criterion.

Solution:

Corrected endurance limit, σ_e = 150 MPa; S_y = 480 MPa S_{ut} = 600 MPa; N = 2

$$P_m = \frac{P_{max} + P_{min}}{2}$$

$$P_a = \frac{P_{max} - P_{min}}{2}$$



$$P_m = 100 \text{ kN}$$

$$P_a = 50 \text{ kN}$$

$$\sigma_m = \frac{100 \times 10^3}{a^2} \text{MPa}$$

$$\sigma_a = \frac{50 \times 10^3}{a^2} \text{MPa}$$

Solution by Goodman's equation,

$$\frac{\sigma_m}{S_{ut}} + \frac{\sigma_a}{\sigma_e} = \frac{1}{N}; \quad 1000\left[\frac{100}{a^2 \times 600} + \frac{50}{150a^2}\right] = \frac{1}{2}$$

$$a^2 = 1000$$

$$a = 31.62 \text{ mm}$$

Solution by yield (Langer's) line equation,

$$\frac{\sigma_m}{S_{yt}} + \frac{\sigma_a}{S_{yt}} = \frac{1}{N}; \qquad 1000\left[\frac{100}{480a^2} + \frac{50}{480a^2}\right] = \frac{1}{2}$$

$$a^2 = 625$$

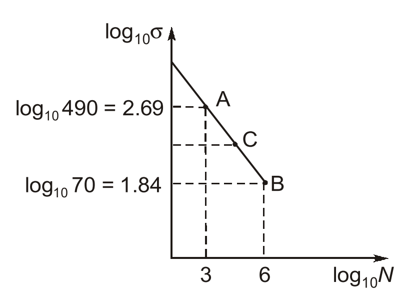
$$a = 25 \text{ mm}$$

Hence final answer by modified **Goodman's criterion is 31.62 mm**

Q.8 A cylindrical shaft is subjected to completely reversed stress of amplitude 100 MPa. Fatigue strength to sustain 1000 cycles is 490 MPa. If the corrected endurance strength is 70 MPa, determine the estimated shaft life.

Solution:

Equation of straight line connecting (3, $\log_{10}$ 490) and (6, $\log_{10}$ 70)



$$\frac{y - 1.8451}{x - 6} = \frac{1.84 - 2.6902}{6 - 3}$$

$$y - 1.8451 = -0.2817\,(x - 6)$$

At $$y = \log_{10} 100 = 2$$

$\Rightarrow$ $$2 - 1.8451 = -0.2817\,(x - 6)$$

$$x = 5.4501$$

$$\log_{10} N = 5.4501, \quad N = \textbf{281914 cycles}$$